





---

RISKY DEBT, JUMP PROCESSES AND SAFETY COVENANTS

by

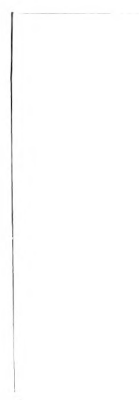
Scott P. Mason

May 1977

WP 997-78

Revised January 1978

---



RISKY DEBT, JUMP PROCESSES AND SAFETY COVENANTS

by

Scott P. Mason

May 1977

WP 997-78

Revised January 1978

I am thankful to Robert Merton for many helpful suggestions. I also thank Sudipto Bhattacharya, E. Philip Jones, and the other participants in the M.I.T. Finance Research Seminar for helpful comments. All errors and omissions are mine.



## INTRODUCTION

Black and Cox [2] analyze the effects of certain bond indenture provisions on the valuation of corporate securities. One specific problem addressed in their paper is the valuation of a risky discount bond in the presence of a safety covenant. A safety covenant is a provision of a bond indenture stipulating that if the value of the firm falls to or below a specified level then the bondholders are entitled to some immediate settlement of their claim on the firm.

The analysis of Black and Cox [2] assumes that the dynamics for the value of the firm can be described by a diffusion process. They suggest that the value of the debt may be altered if the value of the firm follows a jump process since it would then be possible for the value of the firm to reach points below the "barrier" specified by the safety covenant without first passing through it.

This paper presents an approach to this valuation problem and provides some specific results. These results and the valuation methodology are compared to the work of Black and Cox [2]. Lastly, some extensions to this analysis are discussed.

THE VALUATION PROBLEM AND THE BLACK-COX RESULT

Consider a firm with two classes of claims, equity and a single homogeneous class of discount debt, where the bondholders are promised a payment of  $B$  in  $\tau$  time periods. In the event that the promised payment is not made, the entire firm value passes immediately to the bondholders. It is assumed that the bond indenture stipulates that, during the life of the debt, the firm cannot make distributions to the equityholders nor can it issue new senior or equivalent rank claims on the firm.<sup>1</sup>

It is further assumed that the bond indenture specifies a safety covenant of the form;

$$Ce^{-\gamma\tau} < Be^{-r\tau}$$

where  $C$  and  $\gamma$  are constants and  $r$  is the assumed constant instantaneous riskless rate of interest. If the value of the firm should fall to or below this barrier, then the entire firm value passes immediately to the bondholders.

Thus the condition;

$$V \leq Ce^{-\gamma\tau}$$

constitutes a violation of the safety covenant, where  $V$  is the value of the firm with  $\tau$  periods of time remaining in the life of the debt.

Under the additional assumptions:

- A.1) There are no transaction costs, indivisibilities, taxes, bankruptcy costs or agency costs.

---

<sup>1</sup> Black and Cox [2] allow for a constant proportional dividend, which will be omitted here. The analysis to be presented can readily incorporate such a dividend.



- A.2) Every individual acts as if he can buy or sell as much of any security as he wishes at the market price.
- A.3) Short-sales of all assets, with full use of proceeds, is allowed.
- A.4) Trading in assets takes place continuously in time.
- A.5) The dynamics for the value of the firm can be described by a diffusion type stochastic process;

$$\frac{dV}{V} = \alpha dt + \sigma dz$$

where  $\alpha$  is the instantaneous expected rate of return on the firm per unit time,  $\sigma^2$  is the instantaneous variance of the return on the firm per unit time and  $dz$  is a standard Gauss-Wiener process.

Merton [11] demonstrates that any security whose value can be written as a function of the value of the firm and time,  $H(V, \tau)$ , must satisfy;

$$1/2\sigma^2V^2H_{VV} + rVH_V - rH - H_\tau = 0 \quad (1)$$

which is a parabolic partial differential equation where the subscripts denote partial derivatives. Different securities are distinguished by the specification of appropriate initial and boundary conditions.

Black and Cox [2] solve equation (1) with the appended conditions;

$$H(Ce^{-\gamma\tau}, \tau) = Ce^{-\gamma\tau} \quad (1.a)$$

$$H(V, 0) = \text{Min}[V, B] \quad (1.b)$$

by showing that the solution to (1) (1.a) (1.b) is consistent with risk neutral preferences. See Cox and Ross [6]. The boundary condition (1.a) states that the value of the debt equals the value of the firm when the firm

violates the safety covenant. The initial condition (1.b) says that the value of the debt at maturity is the minimum of the value of the firm or the promised payment given that the firm did not violate the safety covenant during the life of the debt. The Black and Cox [2] valuation formula can be represented as;

$$\begin{aligned}
 H(V, \tau) = & Be^{-r\tau} \Phi\left(\frac{\log V - \log B + (r-1/2\sigma^2)\tau}{\sigma\sqrt{\tau}}\right) + V\Phi\left(\frac{\log B - \log V - (r+1/2\sigma^2)\tau}{\sigma\sqrt{\tau}}\right) \\
 & + V\left(\frac{Ce^{-\gamma\tau}}{V}\right)^{\eta+1} \Phi\left(\frac{2\log C - \log B - \log V + (r-2\gamma+1/2\sigma^2)\tau}{\sigma\sqrt{\tau}}\right) \\
 & - Be^{-r\tau} \left(\frac{Ce^{-\gamma\tau}}{V}\right)^{\eta-1} \Phi\left(\frac{2\log C - \log B - \log V + (r-2\gamma-1/2\sigma^2)\tau}{\sigma\sqrt{\tau}}\right)
 \end{aligned} \tag{2}$$

where  $\eta = 2\left(\frac{r-\gamma}{\sigma^2}\right)$  and  $\Phi$  is the unit normal distribution function. The first two terms correspond to the risky discount bond valuation of Merton [11] and the last two terms represent the value of the safety covenant.

JUMP PROCESSES AND A VALUATION METHODOLOGY

It is assumed that jumps or nonlocal movements in the value of the firm are due to the arrival of information concerning the firm.<sup>2</sup> The timing of these information arrivals is random and independent of the impact this information has on the value of the firm. A natural prototype for such a jump process is the Poisson process.

With a Poisson process, the instantaneous probability of information arriving in the time interval  $dt$  is  $\lambda dt$ , where  $\lambda$  is the mean number of arrivals per unit time. The instantaneous probability of no information arriving is  $1 - \lambda dt$ , since the probability of more than one arrival, during the interval  $dt$ , is of an order less than  $dt$ . Given that information has arrived, the impact of that information on the value of the firm is determined by a drawing from a distribution,  $f(Y)$ , where  $Y = V(t + dt)/V(t)$  and  $V(t + dt) - V(t)$  is the change in firm value due solely to the Poisson event. Successive drawings from  $f(Y)$  are independent and all drawings are independent of the timing of the drawing. These firm value dynamics can be formally written as;

$$\frac{dV}{V} = (\alpha - \lambda k)dt + dq$$

where  $\alpha$  is the instantaneous expected rate of return on the firm per unit time,  $dq$  is the Poisson process and  $k = E[Y-1]$ , where  $(Y-1)$  is the random variable percentage change in firm value given the occurrence of a Poisson event and  $E$  is the expectations operator.

---

<sup>2</sup> This brief discussion of jump processes closely follows the work of Merton [12] and Cox and Ross [5].

Thus, the firm will earn a deterministic rate of return,  $(\alpha - \lambda k)$ , during inactive time intervals where no information has arrived;

$$\frac{dV}{V} = (\alpha - \lambda k) dt$$

and will earn a random return during active time intervals where information has arrived;

$$\frac{dV}{V} = (\alpha - \lambda k) dt + (Y - 1)$$

The resulting sample path for  $V$  will be continuous most of the time, with finite jumps, of differing sign and magnitude, occurring at discrete points in time.

Security pricing in the presence of jump processes has been studied by Merton [12] [13] and Cox and Ross [4] [5] [6]. In these cases, the Black-Scholes [3] three asset hedging argument will not result in a pricing equation, analogous to (1), whose solution is consistent with risk neutral preferences. One means of closing these valuation problems is to follow Merton [12] and assume that the jump component in the dynamics for the value of the firm represent diversifiable risk. The risk of the market or the economy may be thought of as a continuous process, but there exist a sufficient number of other securities with contemporaneously independent sources of jump risk so as to make this source of risk diversifiable. This implies that the expected rate of return on the firm is the riskless rate of interest,  $\alpha = r$ .

Writing the value of the debt as a function of the value of the firm and time,  $W(V, \tau)$ , the dynamics for the value of the debt can be written as;

$$\frac{dW}{W} = (\alpha_W - \lambda k_W)dt + dq_W$$

where  $\alpha_W$  is the instantaneous expected rate of return on the debt,  $dq_W$  is a Poisson process with parameter  $\lambda$  and  $k_W = E[Y_W - 1]$ , where  $Y_W = W(VY, \tau)/W(V, \tau)$ . See Merton [12]. The instantaneous expected rate of return,  $\alpha_W$ , on the debt can be represented by the expansion;

$$\alpha_W = \frac{(r - \lambda k) VW_V(V, \tau) - W_T(V, \tau) + \lambda E(W(VY, \tau) - W(V, \tau))}{W(V, \tau)}$$

See Kushner [9]. However, the only source of risk in the return to the debt is  $dq_W$ , which is perfectly functionally dependent on  $dq$ . Thus, since  $dq$  has been assumed to be diversifiable risk, the expected rate of return to the debt must be the riskless rate of interest,  $\alpha_W = r$ .

Under these assumptions, the valuation problem for a risky discount bond in the presence of a safety barrier can be posed as;

$$(r + \lambda)W(V, \tau) - (r - \lambda k) VW_V(V, \tau) + W_T(V, \tau) = \lambda \int W(VY, \tau) f(Y) dY \quad (4)$$

$$W(V, \tau) = V \quad \text{for } V \leq Ce^{-\gamma\tau} \quad (4.a)$$

$$W(V, 0) = \text{Min}[V, B] \quad (4.b)$$

where equation (4) is an integro-differential equation. The boundary condition (4.a) now accounts for the fact that a violation of the safety covenant may result in a settlement less than  $Ce^{-\gamma\tau}$ .

One attack on solving equation (4), subject to conditions (4.a) and (4.b), is suggested by the work of Black and Cox [2]. They demonstrate that their results, (2), are consistent with the approach of computing the expected

discounted value of the payments to the bondholders.<sup>4</sup> In the case of a risky discount bond in the presence of a safety covenant, the pertinent distributions are the first passage time and the defective. The first passage time is the time of the first violation of the safety covenant upon which the debt receives a payment. Here, time  $t$  is measured as the expended life of the debt. The defective is the distribution of the firm value given that the safety covenant has not been violated. At maturity the debt receives the payment  $\text{Min}[V, B]$  which is subject to the defective distribution.

Prabhu [14] demonstrates that the defective distribution,  $D(V, t)$ , associated with a Poisson process, like (3), and a barrier,  $Ce^{-\gamma\tau}$ , must satisfy;

$$D_t(V, t) - (r - \lambda k - \gamma)D_V(V, t) + \lambda D(V, t) = \lambda \int D(VY, t)f(Y)dY \quad (5)$$

$$D(V, 0) = 1 \quad (5.a)$$

If equation (5) can be solved for a given  $f(Y)$ , then the first passage time distribution,  $G(t, V)$ , follows immediately from the identity;

$$G(t, V) = 1 - D(V, t)$$

Few solutions to equation (5) are known. This paper derives the solution for the case where  $f(\cdot)$  is the binomial density.

---

<sup>4</sup> Black and Cox [2] predicate this approach on the argument that their results are invariant to an assumption on preferences; specifically, risk neutrality. Their methodology and the one employed here will be compared in a later section.

THE BINOMIAL CASE

Let;

$$f(Y) = \begin{cases} 1/2 \text{ for } Y = e^{\delta} \\ 1/2 \text{ for } Y = e^{-\delta} \\ 0 \text{ otherwise} \end{cases}$$

where  $\delta$  is a positive constant. The random firm value at time  $t = \tau$  can be written;

$$V(t=\tau) = V \exp((r-\lambda k)\tau + z(\tau)\delta) \quad (6)$$

where  $z(\tau)$  is the sum of a random number,  $n$ , of mutually independent random variables,  $x_i$ :  $i=1,2,\dots,n$ , with the common distribution,

$$g(x) = \begin{cases} 1/2 \text{ for } x = 1 \\ 1/2 \text{ for } x = -1 \\ 0 \text{ otherwise} \end{cases}$$

The random variable  $z(\tau)$  is distributed compound Poisson,  $h(z(\tau))$ , where;

$$h(z(\tau)) = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau} (\lambda\tau)^n}{n!} \{g(x)\}^{n*}$$

where  $n$  is the Poisson distributed random variable number of jumps in  $\tau$  time periods,  $n$  and  $x$  are independent and  $\{g(x)\}^{n*}$  is the  $n$ -fold convolution of  $g(x)$ .

Clearly;

$$\text{prob}[V(t=\tau) = V^*] = \text{prob}[z(\tau) = z^*]$$

where

$$V^* = V \exp((r-\lambda k)\tau + z^*\delta)$$

Now consider the  $\text{prob}[z(\tau) = z^*]$  given exactly  $N$  jumps over the life of the debt;

$$\text{prob}(z(\tau) = z^* | n=N) = \binom{N}{1/2(N+z^*)} (1/2)^N$$

Let;

$$j = \frac{N-z^*}{2}$$

where  $j$  is defined for only those cases where  $\frac{N-z^*}{2}$  is an integer.

$$\text{prob}[z(\tau) = z^* | n=N] = \binom{z^*+2j}{z^*+j} (1/2)^{z^*+2j}$$

Remembering that the occurrence of jumps is governed by a Poisson process, this random walk in  $z(\tau)$  can be "randomized" along the lines of Feller [7];

$$\begin{aligned} \text{prob}[z(\tau) = z^*] &= \sum_{j=0}^{\infty} \frac{e^{-\lambda\tau} (\lambda\tau)^{z^*+2j}}{(z^*+2j)!} \binom{z^*+2j}{z^*+j} (1/2)^{z^*+2j} \\ &= e^{-\lambda\tau} I_{z^*}(\lambda\tau) \end{aligned}$$

where;

$$I_{z^*}(\lambda\tau) = \sum_{j=0}^{\infty} \frac{1}{j! \Gamma(j+z^*+1)} \left( \frac{\lambda\tau}{2} \right)^{2j+z^*}$$

a modified Bessel function of the first kind of order  $z^*$ .

Now consider a barrier,  $Ce^{-\gamma\tau}$ , where  $\gamma = r - \lambda k$ . This case is of interest since its solution will serve as an integral part of the solution to the more general case of arbitrary  $\gamma$ . It is possible to solve for the defective distribution, borrowing from the method of images used to solve the same problem for a Wiener process. First define the integer  $n$ ;



$$n = \left\lfloor \frac{\log V + (r - \lambda k)\tau - \log C}{\delta} \right\rfloor$$

where  $[X]$  is the largest integer smaller than  $X$ . Thus, when  $z(\tau) = n$ , a violation of the safety covenant has occurred.

The proper representation of the defective distribution is;

$$D(V, \tau) = \sum_{z=n}^{\infty} e^{-\lambda \tau} [I_z(\lambda \tau) - I_{z-2n}(\lambda \tau)] \quad (7)$$

The fact that (7) satisfies equation (5) is demonstrated in Appendix A.

The means by which the derivation of (7) is related to the method of images can be seen by considering a single sample path for  $z(\tau)$ . If it is a path that will lead to at least a single violation then  $z(\tau)$  has equaled  $n$  at least once. However, upon arriving at  $n$  for the first time, the distribution of the terminal value of this path is symmetric about  $n$ . Thus the defective distributions can be represented as the difference between the terminal distributions of a process originating at zero and a mirror image of this process originating at  $2n$ .

Defining the integer  $m$ ;

$$m = \left\lfloor \frac{\log V + (r - \lambda k)\tau - \log B}{\delta} \right\rfloor + 1$$

the value of the equity,  $w(V, \tau)$ , and debt,  $W(V, \tau)$ , in the presence of a safety covenant can be written as;

$$w(V, \tau) = \sum_{z=m}^{\infty} e^{-\lambda \tau} (I_z(\lambda \tau) - I_{z-2n}(\lambda \tau)) (V \exp(-\lambda k \tau + z \delta) - B \exp(-r \tau))$$

$$\begin{aligned}
 W(V, \tau) = & \sum_{z=-\infty}^{m-1} e^{-\lambda \tau} (I_z(\lambda \tau)) V \exp(-\lambda k \tau + z \delta) + \sum_{z=m}^{\infty} e^{-\lambda \tau} (I_z(\lambda \tau)) B \exp(-r \tau) \\
 & + \sum_{z=m}^{\infty} e^{-\lambda \tau} I_{z-2n}(\lambda \tau) (V \exp(-\lambda k \tau + z \delta) - B \exp(-r \tau))
 \end{aligned} \tag{8}$$

where the first two terms correspond to the value of the debt in the absence of a safety covenant and the third term represents the value of the safety covenant. The fact that (8) satisfies equation (4) is demonstrated in Appendix B.

Now consider the general case of arbitrary  $\gamma$  values. Define the integers  $n_1, n_2, \dots, n_i$  and the constants  $t_1, t_2, \dots, t_i$  for a given  $\gamma$ ;

$$\begin{aligned}
 n_1 &= \frac{\log V + (r - \lambda k) t_1 - \log C + \gamma(\tau - t_1)}{\delta} \\
 n_2 &= \frac{\log V + (r - \lambda k) t_2 - \log C + \gamma(\tau - t_2)}{\delta} \\
 &\quad \vdots \\
 n_i &= \frac{\log V + (r - \lambda k) t_i - \log C + \gamma(\tau - t_i)}{\delta}
 \end{aligned}$$

which has the effect of transforming the continuous barrier into a step-like function. Clearly, the correct representation of the defective density for  $0 < \tau \leq t_1$  is;

$$d_1(z, \tau; n_1) = e^{-\lambda \tau} [I_z(\lambda \tau) - I_{z-2n_1}(\lambda \tau)]$$

Since the posited firm value dynamics are Markov, the defective density must satisfy the Chapman-Kolmogorov condition. Thus the defective density for

$t_1 < \tau \leq t_2$  is;

$$d_2(z, \tau; n_2) = \sum_{j=n_1}^{\infty} d_1(j, t_1, n_1) e^{-\lambda(\tau-t_1)} (I_{z-j}(\lambda(\tau-t_1)) - I_{z+j-2n_2}(\lambda(\tau-t_1)))$$

And the correct defective density for  $t_{i-1} < \tau \leq t_i$  is;

$$d_i(z, \tau; n_i) = \sum_{j=n_{i-1}}^{\infty} d_{i-1}(j, t_{i-1}; n_{i-1}) e^{-\lambda(\tau-t_{i-1})} (I_{z-j}(\lambda(\tau-t_{i-1})) - I_{z+j-2n_i}(\lambda(\tau-t_{i-1})))$$

Thus, in principle, it is possible to construct the general defective distribution from these defective densities and the analysis would proceed as before.

# DISCUSSION AND COMPARISON OF RESULTS

The comparative statics for  $W(V, \tau)$ , (8), are consistent with those of Black and Cox [2];

$$\frac{\partial W}{\partial V} > 0 \qquad \frac{\partial W}{\partial r} < 0$$

$$\frac{\partial W}{\partial \tau} < 0 \qquad \frac{\partial W}{\partial \lambda} < 0$$

$$\frac{\partial W}{\partial B} > 0$$

where the partial derivative with respect to  $\lambda$  could be interpreted as the behavior of the solution given an increase in the total risk of the firm.<sup>5</sup>

From the definition of  $m$  it is clear that:

$$\sum_{z=m}^{\infty} e^{-\lambda \tau} (I_{z-2n}(\lambda \tau)) (V \exp((r-\lambda k)\tau + z\delta) - B \exp(-r\tau))$$

which is the value of the covenant, is always positive. This agrees with the results of Black and Cox [2].

It is of interest to determine in what sense any results obtained from these Poisson dynamics are "discrete" analogs to the results of Black and Cox [2]. To see this, it will be shown that the characteristic function for the random variable,  $\log [V(t=\tau)/V]$ , will with the proper limiting argument go to that of the normal distribution which governs the same ratio for the dynamics assumed by Black and Cox [2].

---

<sup>5</sup> All partial derivatives are taken in regions which do not effect the value of  $n$  or  $m$ . See Cox and Ross [4].

Let  $\Psi(s)$  be the characteristic function of  $\log(V(t=\tau)/V)$ . It follows from (6);

$$\Psi(s) = [\exp(i(r-\lambda k)\tau s)]\phi(\delta s)$$

where  $\phi(\cdot)$  is the characteristic function of  $z(\tau)$  and  $i=\sqrt{-1}$ .

$$\phi(\delta s) = \exp(-\lambda\tau + \lambda\tau\theta(\delta s))$$

where  $\theta(\cdot)$  is the characteristic function of  $x$ .

$$\theta(\delta s) = E[\exp(ix\delta s)]$$

$$= 1/2[e^{i\delta s} + e^{-i\delta s}] = \cos \delta s$$

Thus;

$$\Psi(s) = \exp[i(r-\lambda k)\tau s - \lambda\tau + \lambda\tau\cos\delta s]$$

Consider the following; let  $\lambda \rightarrow \infty$  as  $\delta \rightarrow 0$  such that  $\lambda\delta^2 = \sigma^2$ , where  $\sigma^2$  is a positive constant.

Evaluating  $k$ ;

$$k = E[Y-1] = 1/2(e^{-\delta} + e^{\delta}) - 1 = \cosh \delta - 1$$

$$k + 1 = \cosh \delta = 1 + \frac{\delta^2}{2!} + \frac{\delta^4}{4!} + \dots$$

$$k = \frac{\delta^2}{2!} + \frac{\delta^4}{4!} + \dots$$

Thus;

$$\lim_{\lambda \rightarrow \infty} \lambda k = 1/2\sigma^2$$

$$\lambda \rightarrow \infty$$

$$\delta \rightarrow 0$$

$$\text{s.t. } \lambda\delta^2 = \sigma^2$$

Recalling;

$$\cos \delta s = 1 - \frac{\delta^2 s^2}{2!} + \frac{\delta^4 s^4}{4!} - \frac{\delta^6 s^6}{6!} + \dots$$

It follows that;

$$\lim_{\lambda \rightarrow \infty} (-\lambda \tau + \lambda \tau \cos \delta s) = -1/2 \sigma^2 \tau s^2$$

$$\lambda \rightarrow \infty$$

$$\delta \rightarrow 0$$

$$\text{s.t. } \lambda \delta^2 = \sigma^2$$

And;

$$\lim_{\lambda \rightarrow \infty} \Psi(s) = \exp[i(\tau - 1/2 \sigma^2) \tau s - 1/2 \sigma^2 \tau s^2]$$

$$\lambda \rightarrow \infty$$

$$\delta \rightarrow 0$$

$$\text{s.t. } \lambda \delta^2 = \sigma^2$$

which is the characteristic function for the normal distribution.

The important difference between this analysis and that of Black and Cox [2] lies in the economics of the necessary assumptions. The results of Black and Cox [2] follow from a strict arbitrage argument, which does not require knowledge of expected rates of return, the equilibrium structure of returns, the existence of an equilibrium, preferences or the existence of securities other than the three used in the hedging argument. Given the assumptions, their result must hold in any economy where agents prefer more wealth to less.

Such is not the case when the dynamics for the value of the firm are subject to jump processes. Some additional assumption must be made in order to close valuation problem in terms of the data. The assumption used in this

paper follows along the lines of the security pricing model developed in Ross [15], and discussed in this context in Merton [12]. The result of this assumption is the identification of the expected rate of return on the firm and its claims as the riskless rate of interest. Note that the use of the Ross model does not require any assumptions on preferences. The valuation formula, (8), follows from a "virtual" arbitrage condition.

This paper was not intended to be an exhaustive treatment of the question of how the value of safety covenants is effected by the presence of jump processes. Several interesting problems remain. One extension of this work could allow the density function governing jump amplitudes to be continuous. An analogous problem to this one has been studied extensively in Collective Risk Theory and few results are known. See Prabhu [14]. Another extension could allow for the dynamics for the value of the firm to be a combined process, involving both Wiener and Poisson processes. Bhattacharya [1] has made some progress on this difficult problem.

Finally, an interesting question is the impact on the pricing of safety covenants due to specification error on the firm value dynamics. More specifically, would the valuation of safety covenants have systematic bias if priced by the Black and Cox result, (2), when the true firm value dynamics contained a jump component? Merton [12] [13] examines this question for stock option pricing.

## REFERENCES

- [1] S. Bhattacharya, "Stock-Price Jumps and Covenant-Induced Safety Barriers: The Perpetuity Case," M.I.T. Dissertation (1977).
- [2] F. Black and J.C. Cox, "Valuing Corporate Securities: Some Effects of Bond Indenture Provision," Journal of Finance (May 1976).
- [3] F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," Journal of Political Economy (May-June 1973).
- [4] J.C. Cox and S.A. Ross, "The Pricing of Options for Jump Processes," Rodney L. White Center Working Paper #2-75, University of Pennsylvania (1975).
- [5] J.C. Cox and S. Ross, "The Valuation of Options for Alternative Stochastic Processes," Journal of Financial Economics (1976).
- [6] J.C. Cox and S.A. Ross, "A Survey of Some New Results in Financial Option Pricing Theory," Journal of Finance (May 1976).
- [7] W. Feller, "Infinitely Divisible Distributions and Bessel Functions Associated with Random Walks," S.I.A.M. Journal of Applied Mathematics (1966).
- [8] J. Ingersoll, Jr., "A Contingent Claims Valuation of Convertible Securities," Center for Mathematical Studies in Business and Economics Report 7628, University of Chicago (July 1976).
- [9] H. Kushner, Stochastic Stability and Control, New York: Academic Press, 1967.
- [10] R.C. Merton, "A Rational Theory of Option Pricing," Bell Journal of Economics and Management Science (Spring 1973).
- [11] R.C. Merton, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," Journal of Finance (May 1974).
- [12] R.C. Merton, "Option Pricing When Underlying Stock Returns Are Discontinuous," Journal of Financial Economics (1976).
- [13] R.C. Merton, "The Impact on Option Pricing of Specification Error in the Underlying Stock Price Returns," Journal of Finance (May 1976).
- [14] N.U. Prabhu, "On the Ruin Problem of Collective Risk Theory," Ann. Math. Statist., (1961).
- [15] S.A. Ross, "The Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory (forthcoming).



## APPENDIX A

The integro-differential equation, (5), in the case where  $\gamma = r - \lambda k$ , reduces to;

$$D_t(V, t) + \lambda D(V, t) = \lambda \int D(VY, t) f(Y) dY \quad (A.1)$$

In the case where  $f(Y)$  is the binomial density, the RHS of (A.1) is equivalent to;

$$\begin{aligned} & \lambda/2 \sum_{z=n+1}^{\infty} e^{-\lambda t} (I_z(\lambda t) - I_{z-2(n+1)}(\lambda t)) \\ & + \lambda/2 \sum_{z=n-1}^{\infty} e^{-\lambda t} (I_z(\lambda t) - I_{z-2(n-1)}(\lambda t)) \end{aligned}$$

The LHS of (A.1);

$$\begin{aligned} & -\lambda D(V, t) + \lambda/2 \sum_{z=n}^{\infty} e^{-\lambda t} (I_{z-1}(\lambda t) + I_{z+1}(\lambda t) \\ & - I_{z-2n-1}(\lambda t) - I_{z-2n+1}(\lambda t)) + \lambda D(V, t) \end{aligned} \quad (B.1)$$

which is seen to be equal to the RHS of (A.1) after collecting terms and changing the summation indices.

Checking the initial condition, (5.a),

$$I_z(0) = 0 \quad \forall z \quad \text{except } z = 0$$

where

$$I_0(0) = 1$$

thus the initial condition is satisfied.

## APPENDIX B

Below it is verified that  $W(V, \tau)$ , (8), satisfies

$$(r+\lambda)W(V, \tau) - (r-\lambda k)VW_V(V, \tau) + W_\tau(V, \tau) = \lambda \int W(VY, \tau) f(Y) dY \quad (B.1)$$

for the case where  $f(Y)$  is the binomial density.

The RHS of (B.1) is equivalent to;

$$\begin{aligned} & \lambda/2 \sum_{z=-\infty}^m e^{-\lambda\tau} I_z(\lambda\tau) V \exp(-\lambda k\tau + z\delta) \\ & + \lambda/2 \sum_{z=m+1}^{\infty} e^{-\lambda\tau} I_z(\lambda\tau) B \exp(-r\tau) \\ & + \lambda/2 \sum_{z=m+1}^{\infty} e^{-\lambda\tau} I_{z-2(n-1)}(\lambda\tau) (V \exp(-\lambda k\tau + z\delta) - B \exp(-r\tau)) \\ & + \lambda/2 \sum_{z=-\infty}^{m-2} e^{-\lambda\tau} I_z(\lambda\tau) V \exp(-\lambda k\tau + z\delta) \\ & + \lambda/2 \sum_{z=m-1}^{\infty} e^{-\lambda\tau} I_z(\lambda\tau) B \exp(-r\tau) \\ & + \lambda/2 \sum_{z=m-1}^{\infty} e^{-\lambda\tau} I_{z-2(n+1)}(\lambda\tau) (V \exp(-\lambda k\tau + z\delta) - B \exp(-r\tau)) \end{aligned}$$

The LHS of (B.1) is as follows;

$$(r+\lambda) \sum_{z=-\infty}^{m-1} e^{-\lambda\tau} I_z(\lambda\tau) (V \exp(-\lambda k\tau + z\delta))$$

$$+ (r+\lambda) \sum_{z=m}^{\infty} e^{-\lambda\tau} I_z(\lambda\tau) B \exp(-r\tau)$$

$$+ (r+\lambda) \sum_{z=m}^{\infty} e^{-\lambda\tau} I_{z-2n}(\lambda\tau) (V \exp(-\lambda k\tau + z\delta) - B \exp(-r\tau))$$

$$- (r-\lambda k) \sum_{z=-\infty}^{m-1} e^{-\lambda\tau} I_z(\lambda\tau) V \exp(-\lambda k\tau + z\delta)$$

$$- (r-\lambda k) \sum_{z=m}^{\infty} e^{-\lambda\tau} I_{z-2n}(\lambda\tau) V \exp(-\lambda k\tau + z\delta)$$

$$- (1+k)\lambda \sum_{z=-\infty}^{m-1} e^{-\lambda\tau} I_z(\lambda\tau) V \exp(-\lambda k\tau + z\delta)$$

$$- (r+\lambda) \sum_{z=m}^{\infty} e^{-\lambda\tau} I_z(\lambda\tau) B \exp(-r\tau)$$

$$- (1+k)\lambda \sum_{z=m}^{\infty} e^{-\lambda\tau} I_{z-2n}(\lambda\tau) V \exp(-\lambda k\tau + z\delta)$$

$$+ (r+\lambda) \sum_{z=m}^{\infty} e^{-\lambda\tau} I_{z-2n}(\lambda\tau) B \exp(-r\tau)$$

$$+ \lambda/2 \sum_{z=-\infty}^{m-1} e^{-\lambda\tau} (I_{z+1}(\lambda\tau) + I_{z-1}(\lambda\tau)) V \exp(-\lambda k\tau + z\delta)$$

$$+ \lambda/2 \sum_{z=m}^{\infty} e^{-\lambda\tau} (I_{z+1}(\lambda\tau) + I_{z-1}(\lambda\tau)) B \exp(-r\tau)$$

$$\begin{aligned}
& + \lambda/2 \sum_{z=m}^{\infty} e^{-\lambda\tau} (I_{z-2n+1}(\lambda\tau) + I_{z-2n-1}(\lambda\tau)) V \exp(-\lambda k\tau + z\delta) \\
& - \lambda/2 \sum_{z=m}^{\infty} e^{-\lambda\tau} (I_{z-2n+1}(\lambda\tau) + I_{z-2n-1}(\lambda\tau)) B \exp(-r\tau)
\end{aligned}$$

The first nine terms sum to zero, while the last four are seen to be equal to the RHS of (B.1). Thus  $W(V, \tau)$ , (8), satisfies (B.1).

Now consider the boundary condition, (4.a),

$$W(V, \tau) = V \quad \text{for } V \leq Ce^{-\gamma\tau}$$

From the definition of  $z(t)$ ;

$$V \leq Ce^{-\gamma\tau} \quad \text{for } z(t) = n$$

$$\begin{aligned}
W(V, \tau) &= \sum_{z=n-m}^{\infty} e^{-\lambda\tau} I_z(\lambda\tau) B \exp(-r\tau) \\
&+ \sum_{z=-\infty}^{n-m-1} e^{-\lambda\tau} I_z(\lambda\tau) V \exp(-\lambda k\tau + z\delta) \\
&+ \sum_{z=n-m}^{\infty} e^{-\lambda\tau} I_z(\lambda\tau) (V \exp(-\lambda k\tau + z\delta) - B \exp(-r\tau)) \\
&= \sum_{z=-\infty}^{\infty} e^{-\lambda\tau} I_z(\lambda\tau) V \exp(-\lambda k\tau + z\delta) \\
&= V
\end{aligned}$$

Finally, consider the initial condition, (4.b);

$$W(V,0) = \text{Min}[V,B]$$

Let  $z(\tau) = z^*$ ;

$$W(V,0) = \sum_{z=-\infty}^{m-z^*-1} I_z(0) \cdot V \exp(z\delta)$$

$$+ \sum_{z=m-z^*}^{\infty} I_z(0) \cdot B$$

$$+ \sum_{z=m-z^*}^{\infty} I_{z-2(n-z^*)}(0) (V \exp(z\delta) - B)$$

If  $z^* \geq m$ , then  $V \geq B$  from the definition of  $m$ . The first and third terms equal zero since  $I_z(0) = 0 \forall z$  except  $z=0$ . The second term equals  $B$  since  $I_0(0) = 1$ . If  $z^* < m$  then  $V < B$ . In this case the second and third terms equal zero and the first term equals  $V$ .





BASE

Date Due

DEC 5 1983

NOV 13 1984

APR 11 1985

JAN 24 1986

Lib-26-67

MAR



HD28.M414 no.994- 78  
Van Breda, M. /A reconciliation of som  
734902 D\*BKS 00055087



3 9080 001 071 007

HD28.M414 no.995- 78  
/A possible design and e  
734900 D\*BKS 00055245



3 9080 001 073 136

HD28.M414 no.996- 78  
Tschoegl, Adri/International barter /  
734898 D\*BKS 00055090



3 9080 001 071 056

HD28.M414 no.997- 78  
Mason, Scott P/Risky debt, jump proces  
735069 D\*BKS 00057798



3 9080 001 104 121

HD28.M414 no.998- 78  
Kobrin, Stephe/Political risk :  
745187 D\*BKS 00143304



3 9080 002 130 471

HD28.M414 no.999- 78  
Holland, Danie/Trends in corporate pro  
735081 D\*BKS 00057788



3 9080 001 103 925

